Algebra 2

4. Polynomial Functions
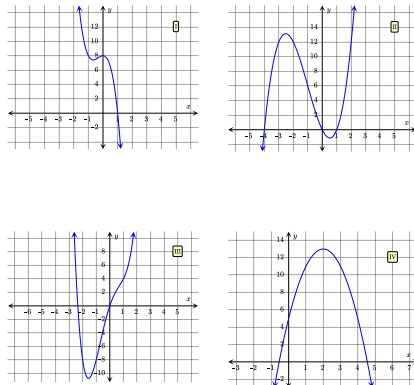
4.1 Sketching Polynomial Functions

Exercises

Find all solutions to exercises via https://mathleaks.com/study/sketching_polynomial_functions or scan the QR code

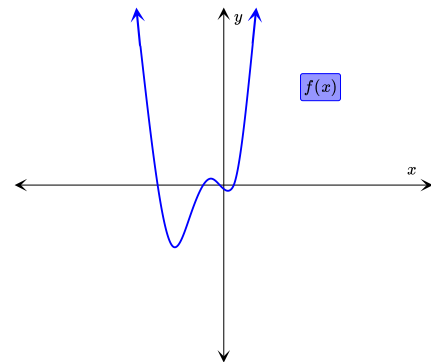**1.1**   Below is presented the graphs of four different polynomial functions.

Use the degree and end behavior to match the polynomial $f(x) = \text{-}4x^3 - 4x^2 + 8$ to its graph.

1.2   Is $\text{-}4x^7 - 3x^6 + 9xy$ a polynomial in one variable? If it is, state its degree and leading coefficient. If it is not, explain why.
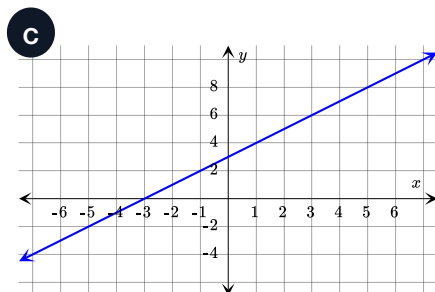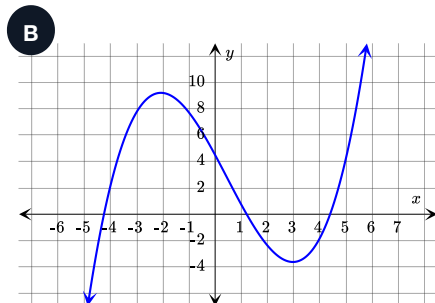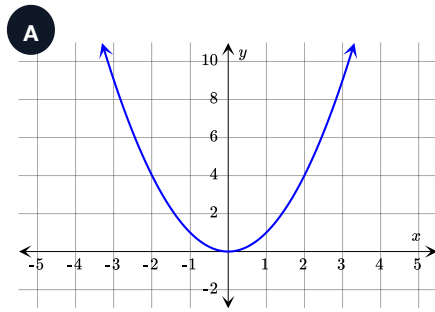
1.3   Presented in the coordinate system is the graph of the polynomial function, $f(x)$.

Using the graph to find out the sign of the leading coefficient and if the polynomial has an odd or an even degree.
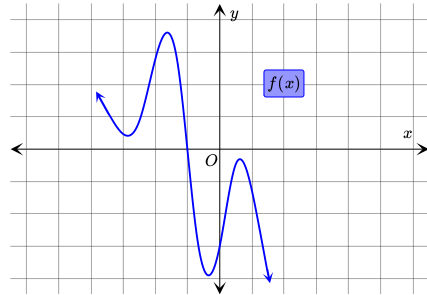
1.4   Is $4a^6 - 3a^3 + \dfrac{5}{a}$ a polynomial in one variable? If it is, state its degree and leading coefficient. If it is not, explain why.

1.5 Determine in what intervals the function is decreasing respectively increasing.

A

B

C

1.6 Is $(d + 5)(3d - 4)$ a polynomial in one variable? If it is, state its degree and leading coefficient. If it is not, explain why.
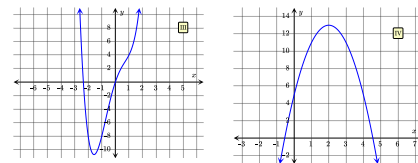
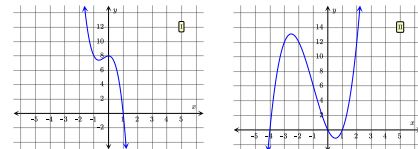**1.7** Presented in the coordinate plane is the graph of the polynomial function $f(x)$.

A Describe the graph's end behavior.

B Does the graph represent an odd-degree or an even-degree polynomial function?

1.8 Below is presented the graphs of four different polynomial functions.

Use the degree and end behavior to match the polynomial $f(x) = x^4 - 3x^2 + 6x$ to its graph.

1.9 What end behavior does the graph of

$$f(x) = 10 - 14x^3 - 3x^5 - 16x^2 - 5x$$

have?

1.10 Is $(5 - 2y)(4 + 3y)$ a polynomial in one variable? If it is, state its degree and leading coefficient. If it is not, explain why.

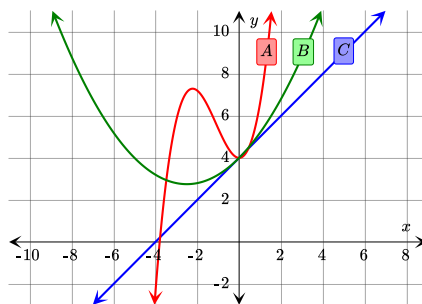**1.11** What end behavior does the graph of

$$f(x) = 8x^5 - 4x^7 + 6x^2$$

have?

1.12 Pair each polynomial

$$p(x) = 0.6x^3 + 2x^2 + 4,$$
$$q(x) = x + 4$$
$$h(x) = 0.2x^2 + x + 4$$

with its corresponding graph.

1.13 Is

$$f(x) = 2x^3 + 3x - \frac{4}{x} + 7x^5 - 1$$

a polynomial function? If it is, write it in standard form and state its degree, type and leading coefficient. If it is not, explain why.

1.14 What end behavior does the graph of

$$f(x) = \text{-}4x^3 + 3x^2 + 2x \text{ have?}$$

1.15 What end behavior does the graph of $f(x) = \text{-}6x^4 + 2x^2$ have?

1.16 Presented in the coordinate system is the graph of the polynomial function, $f(x)$.

Using the graph to find out the sign of the leading coefficient and if the polynomial has an odd or an even degree.

1.17 Presented in the coordinate system is the graph of the polynomial function, $f(x)$.

A Describe the end behavior of $f(x)$.

B Determine whether $f(x)$ is an odd-degree or an even-degree function.

1.18 Which of the functions are polynomial functions?

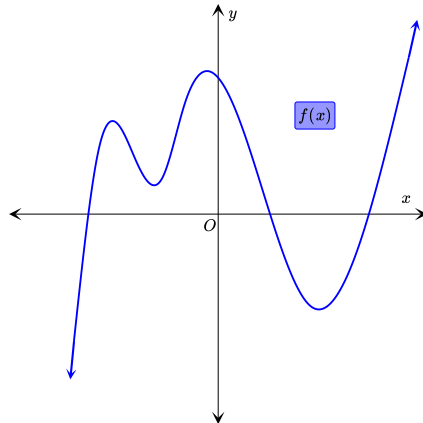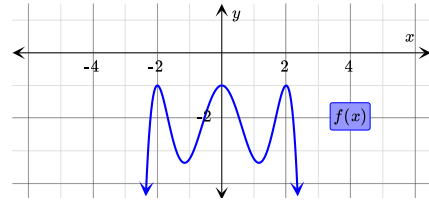- $p(x) = x^5 + 5x^4 - x^2 - 7$

- $q(x) = \dfrac{1}{x}$

- $f(x) = \sqrt{x} + 2$

- $g(x) = x^7 + x^6 + x^{-5} - 1$

- $h(x) = 4x^2 + x - 3$

1.19 Presented in the coordinate plane is the graph of the polynomial function $f(x)$.

A Describe the graph's end behavior.

B Does the graph represent an odd-degree or an even-degree polynomial function?

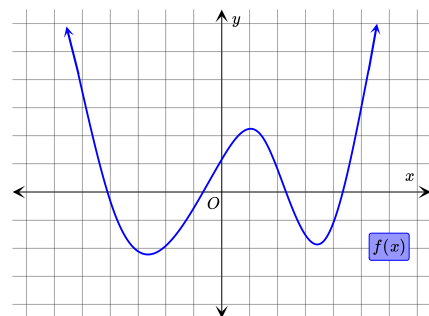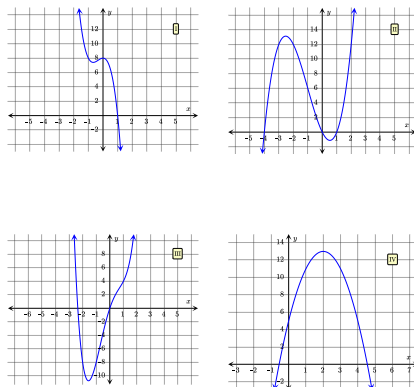**1.20** Presented in the coordinate plane is the graph of the polynomial function $f(x)$.

A Describe the graph's end behavior.

B Does the graph represent an odd-degree or an even-degree polynomial function?

1.21 Below is presented the graphs of four different polynomial functions.

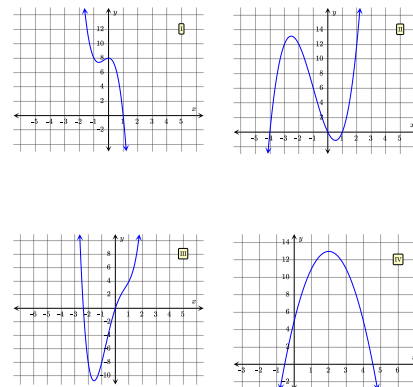Use the degree and end behavior to match the polynomial $f(x) = x^3 + 3x^2 - 4x$ to its graph.

1.23 Below is presented the graphs of four different polynomial functions.

Use the degree and end behavior to match the polynomial $f(x) = \text{-}2x^2 + 8x + 5$ to its graph.
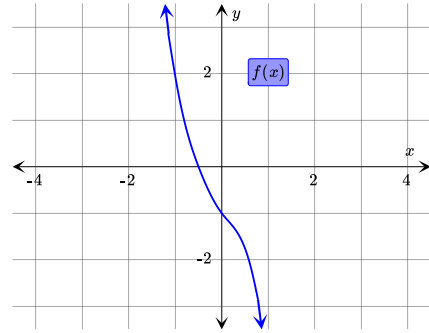
1.22 What end behavior does the graph of
$$f(x) = 5x^7 + 11x^4 - 7x^2 - 4x - 17$$
have?

2.1 Ron-Jon has invented a new card game. For this game he took a traditional deck of cards and replaced the front of each card with an equation for a polynomial function. When he was done the card that used to be jack of clubs looked like this.

$$f(x) =$$

$$2x^4 + 4x^2 - 9\sqrt{x} - 8$$

When he showed his new card game to his sister, Lena-Jon, she told him that he had to replace that card with a new one because she had found an error. Describe and correct the error Lena-Jon found.

2.2 Presented in the coordinate system is the graph of the polynomial function, $f(x)$.

Determine the sign of the leading coefficient and the least possible degree of $f(x)$.

2.3 Presented in the coordinate system is the graph of the polynomial function, $f(x)$.

Determine the sign of the leading coefficient and the least possible degree of $f(x)$.

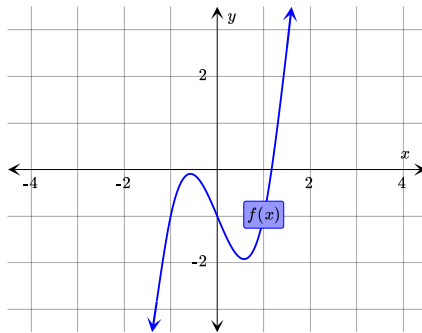**2.4** **A** The graph of the polynomial $p(x)$ of degree 3 has 2 extreme points. Considering the information given, is it possible to know how many zeros the polynomial has?

B The graph for the polynomial $q(x)$ of degree 3 has no extreme points. Considering the information given, is it possible to know how many zeros the polynomial has?

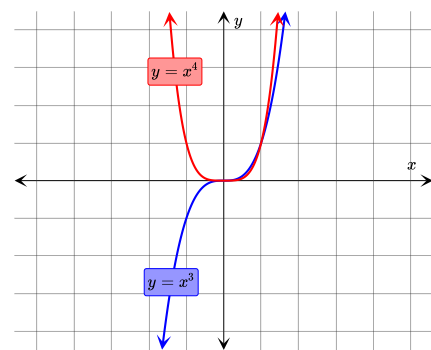**2.5** Presented in the coordinate system is the graph of the polynomial function, $f(x)$.

Determine the sign of the leading coefficient and the least possible degree of $f(x)$.

2.6 Algebraically determine for what values of x the polynomial is decreasing.

A $y = x^2 + 2x - 8$

B $y = \text{-}x^2 + 14x + 15$

2.7 Randall observes the graphs to the polynomial functions $y = x^3$ and $y = x^4$.

He thinks about why the graphs are so different to the left of the origin, but rather similar to the right of the origin. Explain to Randall why the graphs look the way they do!

3.1 The polynomial function $f(x)$ has the following end behavior

$$f(x) \rightarrow \infty \text{ as } x \rightarrow \text{-}\infty$$
$$f(x) \rightarrow \text{-}\infty \text{ as } x \rightarrow \infty$$

Describe the end behavior of $g(x) = \text{-}f(x)$.

3.2 The function $f(x)$ is an even degree polynomial function with the following end behavior.

$$f(x) \to \text{-}\infty \ \text{ as } \ x \to \text{-}\infty$$
$$f(x) \to \text{-}\infty \ \text{ as } \ x \to +\infty$$

Write an equation for $f(x)$ and justify your answer by drawing the graph of your function.

3.3 Some points on the graphs of the polynomial functions $f(x)$ and $g(x)$ are presented in the following table.

x	$f(x)$	$g(x)$
-8	4113	497
-2	21	5
0	1	1
2	13	-3
8	4081	-495

Anna takes a look at the table and says *It seems like the $f(x)$ might be a polynomial with even degree and that $g(x)$ could be a polynomial with odd degree.* Explain how Anna is thinking to come to this conclusion.
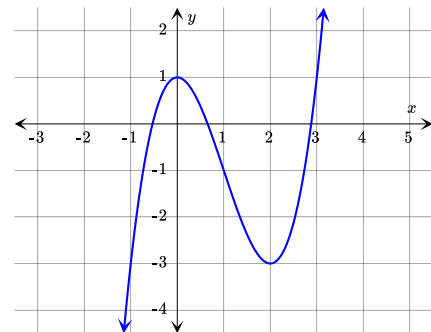
3.4 A polynomial function $p(x)$ is of degree 3 and has 2 different zeros.

A How many solutions does the equation $p(x) = 0$ have?

B How many extrema does the function have?

C Give an example of a polynomial function which matches the description.

3.5 There is a third-degree polynomial $p(x)$ in the coordinate system. Let the constant term be c.

A What is the value of c?

B For what values of c does the polynomial have only one zero?